Problem 1: Unstructured environment with no obstacles

min f(u, x)

1. r.t g(x) = 0 - satisfy vehicle dynamics, initial condition, goal (look ahead distance)

H(x) <= 0 - track, slip angle, steering rate, steering angle, force

f = w1\*t1 + w2\*dist\_to\_center\_line

1. Vehicle Model



Here s is the frenet coordinate along the track; α is the angle of centerline and hence a function of s. p is the design variables and h1 and h2 are the continuous input of acceleration and steering angle. h1 and h2 could be polynomial or spline.

For optimization, there are three variables of interest:



y1 is the integration end time; y2 is the deviation to the goal point and y3 is the average deviation from center line. We’ll need the Jacobian of y w.r.t p. So the problem in ode solver can be formulated as:



A forward integration from t0 to t1 by odeint can compute the value of y.



We don’t want to compute , so we add the ODE to the objective function:



Integrate the underlined term by part:



Substitute and get:

If we set the above two terms to 0, we can avoid calculating 

So  can be computed by integrating backward from t1 -> t0:

 --> Initial condition

 --> An implicit ODE equation

Removing 0 terms from expression:



This concludes the partial derivative calculation of y.

We also need to compute the derivative w.r.t integration end time, which is fairly simple:



The above problem will be addressed in an ‘Adjoint Solver’. The adjoint solver is of general purpose and should allow people to define a 1st order explicit ODE system and any variables with its state. The solver will output state value at t1, variable of interest and its derivative w.r.t parameters. The first version will be implemented in python.